# Existence of Chiral-Asymmetric Zero Modes in the Background of QCD-Monopoles*

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We study topological aspects of the QCD vacuum structure in SU(2) lattice gauge theory with the abelian gauge fixing. The index of the Dirac operator is measured by using the Wilson fermion in the quenched approximation. We find chiral-asymmetric zero modes in background fields dominated by QCD-monopoles without any cooling.

## 1. Topological objects in the QCD vacuum

There are two distinct pictures of the vacuum structure in QCD. One is based on the appearance of color-magnetic monopoles (QCD-monopoles) after performing the abelian gauge fixing [1]. The recent lattice QCD simulations show that the dual Meissner effect brought to the QCD vacuum by QCD-monopole condensation [2]. Hence, color confinement could be regarded as the dual version of the superconductivity. On the other hand, QCD has also classical and non-trivial gauge configurations (instantons) as topological defects in the Euclidean 4-space. As well known, the instanton liquid characterized by a random ensemble of instantons and anti-instanton succeeds in explaining several properties of light hadrons, *e.g.* spontaneous chiral-symmetry breaking (S$\chi$SB) [3].

It seems that QCD-monopoles and instantons are relevant topological objects for the description of each phenomenon. Here, we should mention that QCD-monopoles would play an essential role on non-perturbative features of QCD, which includes S$\chi$SB. This possibility was studied by using the Schwinger-Dyson equation with the gluon propagator in the background of condensed monopoles [4]. The idea of providing S$\chi$SB due to QCD-monopole condensation was supported by the lattice simulations [5,6]. Thus, these results shed new light on the non-trivial relation between QCD-monopoles and instantons. Recently, both analytic and numerical works have shown the existence of the strong correlation between these topological objects [2].

To appreciate further justification for this relation, one should be reminded of the Atiyah-Singer index theorem. The index of the Dirac operator, which corresponds to the number of chiral-asymmetric zero modes, is equal to the topological charge [3]. Thus, instantons can be regarded as important topological objects related to the $\mathrm{U_A}(1)$ anomaly [3]. If the Atiyah-Singer index theorem holds in the background of QCD-monopoles [7], we would find the monopole dominance for the $\mathrm{U_A}(1)$ anomaly. We then study the eigenvalue spectrum of the Dirac operator in background fields dominated by QCD-monopoles in order to examine the existence of chiral-asymmetric zero modes [7].

## 2. Monopole-dominating background field

We generate gauge configurations by using the Monte Carlo simulation on SU(2) lattice gauge theory with the standard Wilson action. The gauge transformation is actually carried out by maximizing the gauge dependent variable $R$;

$$R = \sum_{n,\,\mu} \mathrm{tr}\left\{\sigma_3 U_\mu(n) \sigma_3 U_\mu^\dagger(n)\right\} \quad . \qquad (1)$$

This partial gauge fixing is called as the maximally abelian (MA) gauge [8]. Once the abelian

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gauge fixing is done by this procedure, we factorize the SU(2) link variable $U_\mu$ into the U(1) link variable $u_\mu$ and an off-diagonal part $M_\mu$ as $U_\mu(n) = M_\mu(n) \cdot u_\mu(n)$ where $u_\mu(n) \equiv \exp\{i\sigma_3\theta_\mu(n)\}$ and $M_\mu(n) \equiv \exp\{i\sigma_1 C^1_\mu(n) + i\sigma_2 C^2_\mu(n)\}$. $\theta_\mu$ is the U(1) gauge field and $C^1_\mu$ and $C^2_\mu$ correspond to charged matter fields under a residual U(1) gauge transformation [8].

In order to look for magnetic monopoles in terms of U(1) variables, we consider the product of U(1) link variables around an elementary plaquette, $u_{\mu\nu}(n) = u_\mu(n)u_\nu(n+\hat\mu)u^\dagger_\mu(n+\hat\nu)u^\dagger_\nu(n) = e^{i\sigma_3\theta_{\mu\nu}(n)}$ with the U(1) field strength $\theta_{\mu\nu}(n) \equiv \theta_\nu(n+\hat\mu)-\theta_\nu(n)-\theta_\mu(n+\hat\nu)+\theta_\mu(n)$ [8]. It should be noted that the U(1) plaquette variable is a multiple valued function as the U(1) field strength due to the compactness of the residual U(1) gauge group. Then we can divide the U(1) field strength into two parts as $\theta_{\mu\nu} = \bar\theta_{\mu\nu} + 2\pi N_{\mu\nu}$ where $\bar\theta_{\mu\nu}$ is the regular part defined in $-\pi < \bar\theta_{\mu\nu} \le \pi$ and $N_{\mu\nu} \in \mathbf{Z}$ is the modulo $2\pi$ of $\theta_{\mu\nu}$ [11]. Here, it is known that the abelian dominance for the SU(2) link variable as $U_\mu \simeq u_\mu$ in the MA gauge [2]. In this sense, the Dirac string can be identified by the DeGrand-Toussaint's definition in the compact U(1) lattice gauge theory [11].

The U(1) gauge field can be decomposed into the regular part $\theta^{\rm Ph}_\mu$ and the singular part $\theta^{\rm Ds}_\mu$ as $\theta^L_\mu = \theta^{\rm Ph}_\mu + \theta^{\rm Ds}_\mu$ *in the Landau gauge* $(\partial_\mu\theta^L_\mu = 0)$ [5]. Two parts are respectively defined by

$$\theta^{\rm Ph}_\mu(n) \equiv \sum_{m,\,\lambda} G(n-m)\partial_\lambda\bar\theta_{\lambda\mu}(m) \ , \qquad (2)$$

$$\theta^{\rm Ds}_\mu(n) \equiv 2\pi\sum_{m,\,\lambda} G(n-m)\partial_\lambda N_{\lambda\mu}(m) \qquad (3)$$

where $G(n-m)$ is the lattice Coulomb propagator. It is worth mentioning that the singular part $\theta^{\rm Ds}_\mu$ keeps essential contributions to confining features of the Polyakov loop and finite quark condensate [5].

Next, we define two types of background field, the photon-dominating (monopole-absent) part $U^{\rm Ph}_\mu$ and the monopole-dominating part $U^{\rm Ds}_\mu$ as

$$U_\mu(n) = U^{\rm Ph}_\mu(n)\cdot u^{\rm Ds}_\mu(n) = U^{\rm Ds}_\mu(n)\cdot u^{\rm Ph}_\mu(n) \qquad (4)$$

where $u^i_\mu(n) \equiv \exp\{i\sigma_3\theta^i_\mu(n)\}$ ($i$ = Ph or Ds) [9,10]. It is noted that this definition corresponds to the reconstruction of the corresponding SU(2) variables from $u^i_\mu$ by multiplying the off-diagonal factor in the Landau gauge ($\theta_\mu = \theta^L_\mu + \partial_\mu\phi$) as

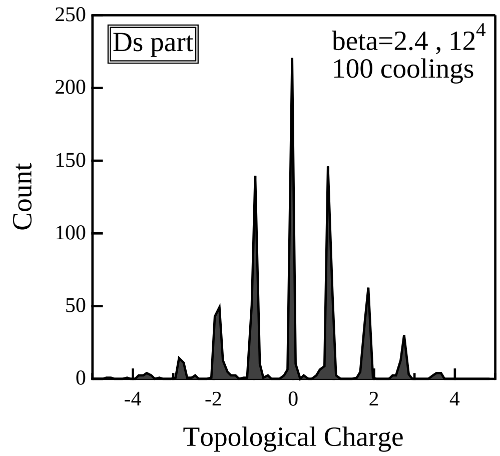


Figure 1. Histogram of topological charges in the background of $U^{\rm Ds}_\mu$ for 1000 configurations.

$$\tilde U^{\,i}_\mu(n) \equiv \tilde M_\mu(n)\exp\{i\sigma_3\theta^{\,i}_\mu(n)\} \qquad (5)$$

where $\tilde U^{\,i}_\mu(n) = d(n)U^{\,i}_\mu(n)d^\dagger(n+\mu)$ and $\tilde M_\mu(n) = d(n)M_\mu(n)d^\dagger(n)$ with $d(n) = e^{i\phi(n)\sigma_3}$ [9,10].

Here, it is worth measuring the topological charge in previously defined backgrounds. As shown in Fig.1, we can observe the topological charge in the background of $U^{\rm Ds}_\mu$ [9,10]. However, non-zero value of the topological charge is not found in the background of $U^{\rm Ph}_\mu$, where instantons seem unable to live [9,10].

## 3. Chiral-asymmetric zero modes

In order to examine the eigenvalue of the Dirac operator $\not{D}$ on the lattice, we adopt the Wilson fermion operator [7]. In the background of $U^{\,i}_\mu$, $\not{D}$ is expressed as

$$\begin{aligned}\not{D}(n,\ m) &= \delta_{n,\,m} - \kappa\sum_\mu \big[(1-\gamma_\mu)U^{\,i}_\mu(n)\delta_{n+\hat\mu,\,m} \\ &\qquad +(1+\gamma_\mu)U^{\,i\,\dagger}_\mu(n-\hat\mu)\delta_{n-\hat\mu,\,m}\big] \qquad (6)\end{aligned}$$

where $\kappa$ is the hopping parameter. The operator $\not{D}$ loses a feature as the hermitian opera-

tor owing to the discretization of the space-time. However, we can easily see that $\gamma_5 \not{D}$ is a hermitian matrix. Then, the existence of chiral-asymmetric zero modes can be identified by the zero-line crossing in the eigenvalue spectrum of $\gamma_5 \not{D}$ through the variation of $\kappa$ [12].

We measure the eigenvalue of the operator $\gamma_5 \not{D}$ in the background of $U_\mu^i$ ($i =$ Ph or Ds) and also in the original gauge field $U_\mu$ for 32 independent configurations *without any cooling*. Fig.2(a)-(c) show the low-lying spectra in each background, which is based on the same gauge configuration, as typical examples. We can see that 2 zero modes exist in the monopole dominating background (Ds part) and the original SU(2) gauge field. This remarkable coincidence for the number of the zero modes is not well identified in 6 configurations, but is confirmed in the rest 26 configurations. On the other hand, we can never find the corresponding zero modes in the monopole-absent background (Ph part) within 32 configurations.

In conclusion, we have investigated the eigenvalue spectrum for the Dirac operator in the background of the monopole-absent (Ph) part, the monopole-dominating (Ds) part and the original gauge field on an $8^4$ lattice at $\beta = 2.4$ by using the Lanczos algorithm. We have found the existence of chiral-asymmetric zero modes in the background of QCD-monopoles, where instantons could survive.

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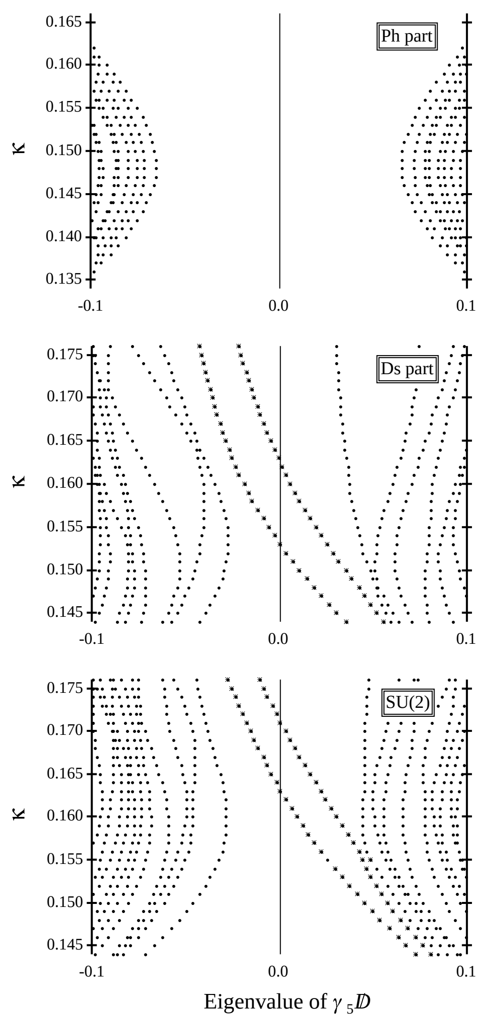


Figure 2. Typical examples of the low-lying spectra of $\gamma_5 \not{D}$ in the background of (a) $U_\mu^{\rm Ph}$, (b) $U_\mu^{\rm Ds}$ and (c) $U_\mu$.